Signatures of nonlinear cavity optomechanics in the weak coupling regime

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We identify signatures of the intrinsic nonlinear interaction between light and mechanical motion in cavity optomechanical systems. These signatures are observable in the resolved sideband regime even when the cavity linewidth exceeds the optomechanical coupling rate. A strong laser drive red-detuned by twice the mechanical frequency from the cavity resonance frequency makes two-phonon processes resonant, which leads to a nonlinear version of optomechanically induced transparency. This effect provides a new method of measuring the average phonon number of the mechanical oscillator. Furthermore, we show that if the strong laser drive is detuned by half the mechanical frequency, optomechanically induced transparency also occurs due to resonant two-photon processes. The cavity response to a second probe drive is in this case nonlinear in the probe power. These effects should be observable with optomechanical coupling strengths that have already been realized in experiments.

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Introduction. Spectacular advances in the quality factor of nano- and micro-mechanical oscillators and their rapidly increasing coupling to optical and microwave resonators have given rise to remarkable progress in the field of cavity optomechanics [1, 2]. This has enabled cooling of mechanical oscillators to their motional quantum ground-state [3, 4] and observations of optomechanically induced transparency [5–7], quantum zero-point motion [8, 9], as well as squeezed light and radiation pressure shot noise [10–12].

The interaction between light and mechanical motion due to radiation pressure is intrinsically nonlinear. While several theoretical studies of the single-photon strong-coupling regime have been reported recently [13–20], most realizations of cavity optomechanics are still in the weak coupling limit where the coupling rate is much smaller than the cavity linewidth. Experiments to date have relied on strong optical driving, which enhances the coupling at the expense of making the effective interaction linear. Realizations that show promise for entering the strong coupling regime include the use of cold atoms [10], superconducting circuits [6], microtoroids [21], or silicon-based optomechanical crystals [4]. In the latter, a ratio between the coupling rate and the cavity linewidth of 0.007 has been reported [22], and improvements seem feasible. To enter the nonlinear regime of cavity optomechanics is of great interest, since it is only then that the internal dynamics can lead to true non-classical behaviour [23].

In this article, we study corrections to linearized optomechanics and identify signatures of the intrinsic nonlinear coupling that are observable even with a relatively weak optomechanical coupling. The nonlinear effects we discuss come about due to the presence of a strong optical drive. We show that if this drive is detuned by *twice* the mechanical frequency from the cavity resonance frequency, two-phonon processes become resonant. This gives rise to a nonlinear version of optomechanically induced transparency (OMIT). OMIT has been well studied in linearized optomechanics [24] and is analogous to electromagnetically induced transparency in atomic systems. We point out that the two-phonon induced OMIT enables a precise measurement of the effective average phonon number of the mechanical oscillator. This provides an alternative to sideband thermometry [8, 9, 25, 26]. Furthermore, we show that OMIT also occurs if the drive is detuned by *half* the mechanical frequency due to two-photon resonances, and the cavity response to a second probe drive is then nonlinear in probe power. We expect these effects to be observable for coupling strengths that have already been realized in experiments. Their observation would verify the intrinsic nonlinearity of the optomechanical interaction and thus open up the possibility of generating non-classical states.

To relate to previous work, we note that a two-phonon induced transparency [27] can also occur in optomechanical systems where the cavity frequency depends quadratically on the position of the mechanical oscillator [28]. In addition, the effect of ordinary linear OMIT on higher-order optical sidebands was studied in Ref. [29].

Model. We consider a standard optomechanical system described by the Hamiltonian $\hat{H} = \hat{H}_{\rm sys} + \hat{H}_{\rm pump}$. The system Hamiltonian is

$$\hat{H}_{\rm sys} = \hbar\omega_{\rm r}\hat{a}^\dagger\hat{a} + \hbar\omega_{\rm m}\hat{c}^\dagger\hat{c} + \hbar g\left(\hat{c}+\hat{c}^\dagger\right)\left(\hat{a}^\dagger\hat{a} - |\bar{a}_{\rm p}|^2\right), \quad (1)$$

where $\hat{a}$ ($\hat{c}$) is the photon (phonon) annihilation operator, $\omega_{\rm r}$ ($\omega_{\rm m}$) the bare cavity (mechanical) resonance frequency, and g the single-photon coupling rate. The mechanical position operator is $\hat{x} = x_{\rm zpf}\hat{z}$, where $\hat{z} = \hat{c}+\hat{c}^\dagger$, $x_{\rm zpf} = \sqrt{\hbar/2m\omega_{\rm m}}$ is the size of the zero-point fluctuations, and m is the effective mass of the oscillator. The constant $|\bar{a}_{\rm p}|^2$ is included for convenience and simply shifts the equilibrium position of the mechanical oscillator. The cavity mode is driven by a laser at the frequency $\omega_{\rm p}$. This drive will be referred to as the pump, and is described by the Hamiltonian $\hat{H}_{\rm pump} = i\hbar(e^{-i\omega_{\rm p}t}\Omega_{\rm p}\hat{a}^\dagger - {\rm h.c.})$.

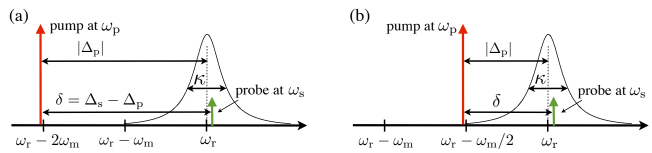

FIG. 1. (color online). Depiction of the setup when the pump detuning is (a) $\Delta_\mathrm{p} \approx -2\omega_\mathrm{m}$ and (b) $\Delta_\mathrm{p} \approx -\omega_\mathrm{m}/2$. Note the difference in scale between (a) and (b). Δ_p and Δ_s are the detunings of the pump and probe lasers from the bare cavity resonance frequency ω_r, whereas $\tilde{\Delta}_\mathrm{p}$ and $\tilde{\Delta}_\mathrm{s}$ are the detunings from the *effective* cavity resonance frequency.

Let us move to a frame rotating at the pump frequency ω_p and perform a displacement transformation, such that $\hat{a}(t) \to e^{-i\omega_\mathrm{p} t}\left[\bar{a}_\mathrm{p} + \hat{a}(t)\right]$. We define $\Delta_\mathrm{p} = \omega_\mathrm{p} - \omega_\mathrm{r} \neq 0$ as the pump detuning from the cavity resonance and choose $\bar{a}_\mathrm{p} = i\Omega_\mathrm{p}/\Delta_\mathrm{p}$. This results in the Hamiltonian $\hat{H} = \hat{H}_0 + \hat{H}_1$, where

$$\hat{H}_0 = -\hbar\Delta_\mathrm{p}\hat{a}^\dagger\hat{a} + \hbar\omega_\mathrm{m}\hat{c}^\dagger\hat{c} + \hbar G\left(\hat{c} + \hat{c}^\dagger\right)\left(\hat{a} + \hat{a}^\dagger\right), \quad (2)$$

$$\hat{H}_1 = \hbar g\left(\hat{c} + \hat{c}^\dagger\right)\hat{a}^\dagger\hat{a}. \quad (3)$$

We have introduced $G = g\bar{a}_\mathrm{p}$ and assumed, without loss of generality, that $\bar{a}_\mathrm{p}$ is real. The coupling G is enhanced by the square root of the average cavity photon number compared to g and provides a bilinear coupling between photons and phonons. This coupling has been well studied, and it is known to give rise to effects such as sideband cooling [3, 4, 30, 31] and OMIT [5–7, 24].

Identifying resonant nonlinear terms. The bilinear Hamiltonian $\hat{H}_0$ with $\Delta_\mathrm{p} < 0$ simply describes two linearly coupled harmonic oscillators. By a symplectic transformation, we can express $\hat{H}_0$ in terms of new operators $\hat{A}$ and $\hat{C}$, which are annihilation operators for the normal mode excitations of the system. These excitations are in general superpositions of photonic and phononic degrees of freedom. Up to a constant, the Hamiltonian becomes

$$\hat{H}_0 = -\hbar\tilde{\Delta}_\mathrm{p}\hat{A}^\dagger\hat{A} + \hbar\tilde{\omega}_\mathrm{m}\hat{C}^\dagger\hat{C}\,. \quad (4)$$

We will assume that $G/\omega_\mathrm{m} \ll 1$, and that the pump frequency ω_p does *not* coincide with the sideband frequencies $\omega_\mathrm{r} \pm \omega_\mathrm{m}$, but rather that $|\omega_\mathrm{m} \pm \Delta_\mathrm{p}|$ is on the order of ω_m. See Fig. 1 for an overview of the frequencies involved. In this case, the operator $\hat{A}$ describes excitations that are photon-like, while $\hat{C}$ describes phonon-like excitations. To second order in G/ω_m, we get $\hat{A} = [1+2\lambda_+\lambda_-\rho/(1-\rho^2)]\hat{a} - \lambda_+\hat{c} - \lambda_-\hat{c}^\dagger - \lambda_+\lambda_-\rho\,\hat{a}^\dagger$ and $\hat{C} = [1+2\lambda_+\lambda_-\rho/(1-\rho^2)]\hat{c} + \lambda_+\hat{a} - \lambda_-\hat{a}^\dagger + \lambda_+\lambda_-\rho^{-1}\,\hat{c}^\dagger$ when we define $\rho = \omega_\mathrm{m}/\Delta_\mathrm{p}$ and $\lambda_\pm = G/(\Delta_\mathrm{p} \pm \omega_\mathrm{m})$. The normal-mode frequencies are $\tilde{\Delta}_\mathrm{p} = \Delta_\mathrm{p}\left(1 - 2\lambda_+\lambda_-\rho\right)$ and $\tilde{\omega}_\mathrm{m} = \omega_\mathrm{m}\left(1 + 2\lambda_+\lambda_-\rho^{-1}\right)$.

We can now rewrite the Hamiltonian $\hat{H}_1$ in terms of the normal-mode operators $\hat{A}$ and $\hat{C}$, which results in multiple terms. However, since $g/\kappa, G/\omega_\mathrm{m} \ll 1$, we only retain terms of nonzero order in G/ω_M if they are resonant. We consider two different pump detunings. First, if $\Delta_\mathrm{p} \sim -2\omega_\mathrm{m}$, we find $\hat{H}_1 = \hbar\, g\left(\hat{C} + \hat{C}^\dagger\right)\hat{A}^\dagger\hat{A} + \hat{H}_{1,\mathrm{res}}$, where the resonant terms are

$$\hat{H}_{1,\mathrm{res}} = -\hbar g\frac{G}{\omega_\mathrm{m}}\left(\hat{A}^\dagger\hat{C}^2 + \hat{C}^{\dagger 2}\hat{A}\right)\,. \quad (5)$$

This describes processes where one photon-like excitation is created and two phonon-like excitations are destroyed, and vice versa. On the other hand, if $\Delta_\mathrm{p} \sim -\omega_\mathrm{m}/2$, the resonant terms are

$$\hat{H}_{1,\mathrm{res}} = -\frac{8}{3}\hbar g\left(\frac{G}{\omega_\mathrm{m}}\right)^2\left(\hat{A}^{\dagger 2}\hat{C} + \hat{C}^\dagger\hat{A}^2\right)\,, \quad (6)$$

which describes processes where two photon-like excitations are created and one phonon-like excitation is destroyed, and vice versa. The Hamiltonian (4) combined with (5) or (6) gives rise to new effects beyond standard linearized optomechanics, which we study below.

Equations of motion. We include dissipation and additional drives by input-output theory [32, 33]. The cavity and mechanical energy decay rates are κ and γ, respectively. We assume that $\kappa \gg \gamma$ and that the system is in the resolved sideband regime where $\omega_\mathrm{m} > \kappa$. Note that in the presence of dissipation, the amplitude $\bar{a}_\mathrm{p} = \Omega_\mathrm{p}\chi_\mathrm{r}(\Delta_\mathrm{p})$, where the cavity susceptibility is defined as $\chi_\mathrm{r}(\omega) = (\kappa/2 - i\omega)^{-1}$. The drive strength Ω_p is related to the laser power P_p through $|\Omega_\mathrm{p}|^2 = \kappa_\mathrm{ext}P_\mathrm{p}/(\hbar\omega_\mathrm{p})$, where $\kappa_\mathrm{ext} \leq \kappa$ is the decay rate of the port through which the cavity couples to the drive.

The quantum Langevin equations are [34]

$$\dot{\hat{a}} = -\left(\frac{\kappa}{2} - i\Delta_\mathrm{p}\right)\hat{a} - i(G + g\hat{a})(\hat{c} + \hat{c}^\dagger) + \sqrt{\kappa}\,\hat{a}_\mathrm{in} \quad (7)$$

$$\dot{\hat{c}} = -\left(\frac{\gamma}{2} + i\omega_\mathrm{m}\right)\hat{c} - iG(\hat{a} + \hat{a}^\dagger) - ig\hat{a}^\dagger\hat{a} + \sqrt{\gamma}\,\hat{c}_\mathrm{in}. \quad (8)$$

We now introduce a weak second optical drive, the probe, with frequency ω_s close to the cavity resonance frequency ω_r. We define the optical input operator in Eq. (7) by $\sqrt{\kappa}\,\hat{a}_\mathrm{in}(t) = e^{-i\delta t}\Omega_\mathrm{s} + \sqrt{\kappa_\mathrm{ext}}\hat{\xi}_\mathrm{ext}(t) + \sqrt{\kappa_\mathrm{int}}\hat{\xi}_\mathrm{int}(t)$, where κ_int is the internal decay rate, $\kappa_\mathrm{ext} + \kappa_\mathrm{int} = \kappa$ and $\delta = \omega_\mathrm{s} - \omega_\mathrm{p}$ is the frequency difference between the two drives. The frequency Ω_s is related to the probe power P_s by $|\Omega_\mathrm{s}|^2 = \kappa_\mathrm{ext}P_\mathrm{s}/(\hbar\omega_\mathrm{s})$. The vacuum noise operators $\hat{\xi}_\mathrm{ext}$ obey $\langle\hat{\xi}_\mathrm{ext}(t)\hat{\xi}^\dagger_\mathrm{ext}(t')\rangle = \delta(t - t')$ and $\langle\hat{\xi}^\dagger_\mathrm{ext}(t)\hat{\xi}_\mathrm{ext}(t')\rangle = 0$ and similarly for $\hat{\xi}_\mathrm{int}$. The mechanical oscillator is not driven, but coupled to a thermal bath, such that the mechanical input operator obey $\langle\hat{c}_\mathrm{in}(t)\hat{c}^\dagger_\mathrm{in}(t')\rangle = (n_\mathrm{th} + 1)\delta(t - t')$ and $\langle\hat{c}^\dagger_\mathrm{in}(t)\hat{c}_\mathrm{in}(t')\rangle = n_\mathrm{th}\delta(t - t')$, where n_th is the bath temperature expressed as a number of quanta. We will solve Eqs. (7) and (8) perturbatively in the single-photon coupling g [35]. The coupling G cannot be treated perturbatively, but we will exploit the fact that $G/\omega_\mathrm{m} \ll 1$.

The presence of two optical drives gives rise to a beat note in the optical intensity at frequency δ. This produces an off-resonant drive on the mechanical oscillator, as we have assumed $\delta \neq \omega_\mathrm{m}$. To avoid parametric instability, we must ensure that the cavity frequency modulations due to the coherent motion induced by this beat note are much smaller than

the cavity linewidth, i.e. $gG|\Omega_\mathrm{s}|/(\kappa\omega_\mathrm{m}) \ll \kappa$. This is easily fulfilled for a weak probe drive with $|\Omega_\mathrm{s}| \lesssim \kappa$ along with our assumptions of $G/\omega_\mathrm{m}, g/\kappa \ll 1$. Note that other instabilities can also arise [36] and must be avoided.

We can express Eqs. (7) and (8) in terms of the normal mode operators $\hat{A}$ and $\hat{C}$. This still gives equations with linear coupling terms whenever dissipation is present. However, in the extreme resolved sideband limit $\kappa/\omega_\mathrm{m} \ll 1$, they simplify to

$$\dot{\hat{A}} = -\left(\frac{\kappa}{2} - i\tilde{\Delta}_\mathrm{p}\right)\hat{A} + \frac{i}{\hbar}[\hat{H}_1, \hat{A}] + \sqrt{\kappa}\,\hat{a}_\mathrm{in} \tag{9}$$

$$\dot{\hat{C}} = -\left(\frac{\tilde{\gamma}}{2} + i\tilde{\omega}_\mathrm{m}\right)\hat{C} + \frac{i}{\hbar}[\hat{H}_1, \hat{C}] + \sqrt{\tilde{\gamma}}\,\tilde{c}_\mathrm{in}. \tag{10}$$

The effective mechanical linewidth is $\tilde{\gamma} = \gamma - \nu\kappa$ where $\nu \equiv 4\lambda_+\lambda_-\rho/(1-\rho^2) < 0$ for $\Delta_\mathrm{p} < 0$. The effective frequencies $\tilde{\omega}_\mathrm{m}$ and $\tilde{\Delta}_\mathrm{p}$ were defined above. Note that $|\nu| \sim (G/\omega_\mathrm{m})^2 \ll 1$ such that the effective mechanical linewidth is still small compared to the cavity linewidth, i.e. $\tilde{\gamma} \ll \kappa$. The effective mechanical noise operator is defined by $\sqrt{\tilde{\gamma}}\,\tilde{c}_\mathrm{in} = \sqrt{\tilde{\gamma}_0}\,\hat{c}_\mathrm{in} + \sqrt{\kappa}(\lambda_+\hat{\xi} + \lambda_-\hat{\xi}^\dagger)$ when ignoring the beat note and defining $\sqrt{\kappa}\,\hat{\xi} \equiv \sqrt{\kappa_\mathrm{ext}}\hat{\xi}_\mathrm{ext} + \sqrt{\kappa_\mathrm{int}}\hat{\xi}_\mathrm{int}$. Its autocorrelation properties are the same as for $\hat{c}_\mathrm{in}$, but with n_th replaced by the effective phonon number $n_\mathrm{m} = (\gamma n_\mathrm{th} + \kappa\lambda_-^2)/\tilde{\gamma}$. The last term in n_m describes heating due to radiation pressure shot noise [30, 31].

Two-phonon induced transparency. We start by focusing on the case of a pump detuned by twice the mechanical frequency, $\tilde{\Delta}_\mathrm{p} = -2\tilde{\omega}_\mathrm{m}$, where two-phonon processes are resonant according to Eq. (5). Such processes have been studied before for systems with so-called quadratic optomechanical coupling [28], and it has been shown that they can lead to OMIT [27] much in the same way as single-phonon processes do with ordinary linear optomechanical coupling [24]. We will now see that two-phonon induced transparency can also occur in the case of linear optomechanical coupling.

By solving Eqs. (9) and (10) perturbatively in the single-photon coupling g and transforming back to the original operators, we calculate the optical coherence $\langle\hat{a}(t)\rangle$ at frequencies close to the resonance frequency. Defining the probe beam detuning by $\Delta_\mathrm{s} = \omega_\mathrm{s} - \omega_\mathrm{r}$ and the effective detuning $\tilde{\Delta}_\mathrm{s} = \Delta_\mathrm{s} - \Delta_\mathrm{p} + \tilde{\Delta}_\mathrm{p}$, we find $\langle\hat{a}(t)\rangle = e^{-i\delta t}\bar{a}_\mathrm{s}$ where

$$\bar{a}_\mathrm{s} = \bar{a}_{\mathrm{s},0}\left(1 - \alpha - \frac{2g_1^2\chi_\mathrm{r}(\tilde{\Delta}_\mathrm{s})\langle\hat{z}_0^2\rangle}{\tilde{\gamma} - i(\tilde{\Delta}_\mathrm{s} - \tilde{\Delta}_\mathrm{p} - 2\tilde{\omega}_\mathrm{m})}\right) \tag{11}$$

and $\bar{a}_{\mathrm{s},0} = \Omega_\mathrm{s}\chi_\mathrm{r}(\tilde{\Delta}_\mathrm{s})$. We have introduced $g_1 = gG/\omega_\mathrm{m}$, which is the effective coupling rate appearing in Eq. (5). The first term in Eq. (11) is the response of an empty cavity. The second term, given by $\alpha = g^2\chi_\mathrm{r}(\tilde{\Delta}_\mathrm{s})[(n_\mathrm{m}+1)\chi_\mathrm{r}(-\omega_\mathrm{m}) + n_\mathrm{m}\chi_\mathrm{r}(\omega_\mathrm{m})]$, does not contain a sharp feature and results from Raman scattering of probe photons to the sideband frequencies $\omega_\mathrm{s} \pm \tilde{\omega}_\mathrm{m}$. The last term gives rise to a narrow dip of width $2\tilde{\gamma}$ in the coherent amplitude as well as a group delay of the input signal. This is analogous to the well-studied case of linear OMIT for pump detuning $\tilde{\Delta}_\mathrm{p} = -\tilde{\omega}_\mathrm{m}$. In the case of $\tilde{\Delta}_\mathrm{p} = -2\tilde{\omega}_\mathrm{m}$, however, the effect is *not* due to coherent driving of the mechanical oscillator. The size of the effect rather depends on the average mechanical *fluctuations* through $\langle\hat{z}_0^2\rangle \equiv (2n_\mathrm{m}+1)$. This is also the case for two-phonon induced transparency with quadratic optomechanical coupling [27]. Note that the average displacement can be increased by mechanically driving the oscillator.

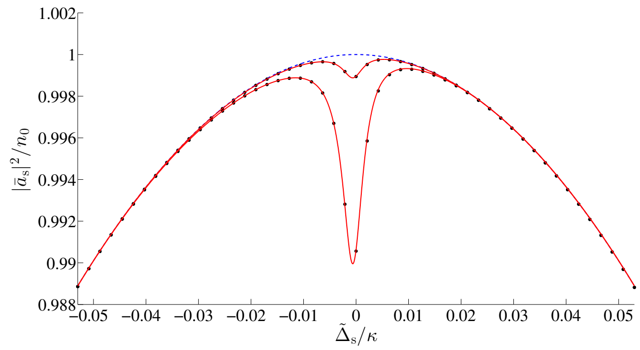

FIG. 2. (color online). The cavity response $|\bar{a}_\mathrm{s}|^2$ in Eq. (11) in units of $n_0 = (2\Omega_\mathrm{s}/\kappa)^2$ in the case of pump detuning $\tilde{\Delta}_\mathrm{p} = -2\tilde{\omega}_\mathrm{m}$. The parameters are $G/\omega_\mathrm{m} = 0.05$, $\kappa/\omega_\mathrm{m} = 0.1$, $\Omega_\mathrm{s}/\kappa = 0.01$, $n_\mathrm{th} = 1$, and $\omega_\mathrm{m}/\gamma = 10^5$. *Upper solid:* $g/\kappa = 0.01$. *Lower solid:* $g/\kappa = 0.03$. *Dashed:* $g/\kappa = 0$. *Dots:* Numerical results.

If the system is not in the extreme resolved sideband limit $\kappa/\omega_\mathrm{m} \ll 1$, additional terms must be included in Eqs. (9) and (10). This leads to the same expression as in (11), but with the changes $g_1^2 \to i\omega_\mathrm{m}g^2G^2|\chi_\mathrm{r}(\omega_\mathrm{m})|^2\chi_\mathrm{r}(-\omega_\mathrm{m})$, and $\langle\hat{z}_0^2\rangle \to \langle\hat{z}_0^2\rangle - \kappa/(2i\omega_\mathrm{m})$. The latter correction is due to optomechanical correlations induced by the radiation pressure shot noise [37]. In addition, the effective parameters describing the mechanical oscillator are also adjusted if κ/ω_m is not negligible [38].

The cavity response $|\bar{a}_\mathrm{s}|^2$ to the probe drive is plotted in Fig. 2 for $g/\kappa = 0.01$ and 0.03. The dip in $|\bar{a}_\mathrm{s}|^2$ corresponds to a dip in either transmission or reflection of the probe depending on the experimental setup.

The result (11) provides a new way of measuring the average phonon number of the mechanical oscillator. To see this in an easy way, let us assume $\kappa/\omega_\mathrm{m} \ll 1$ and $\tilde{\Delta}_\mathrm{p} = -2\tilde{\omega}_\mathrm{m}$, and that the mechanical oscillator is not driven. We define the size of the dip $d \leq 1$ at $\tilde{\Delta}_\mathrm{s} = 0$ as $d \equiv 1 - \mu|\bar{a}_\mathrm{s}/\bar{a}_{\mathrm{s},0}|^2 = 2\mu K_1(2n_\mathrm{m}+1)$ to lowest order in g_1, where $K_1 = 4g_1^2/(\kappa\tilde{\gamma})$ is the *effective* single-photon cooperativity and $1/\mu = 1 - 2\mathrm{Re}\,\alpha = 1 - 2g^2|\chi_\mathrm{r}[\omega_\mathrm{m}]|^2(2n_\mathrm{m}+1) \approx 1$. In the limit where the optical broadening of the mechanical linewidth is significant, i.e. $\kappa(G/\omega_\mathrm{m})^2 \gg \gamma$, the size of the dip becomes $d = 9(g/\kappa)^2(2n_\mathrm{m}+1)$ [39] and the phonon number can easily be measured if g/κ is known. This could serve as an alternative to regular sideband thermometry [8, 9, 25, 26]. Finally, we note that while the two-phonon OMIT is a classical effect, its presence in the low-temperature limit $n_\mathrm{m} \to 0$ is solely due to quantum zero-point fluctuations of the mechanical oscillator.

Two-photon induced transparency. We now consider the case of the pump drive detuned by half the mechanical fre-

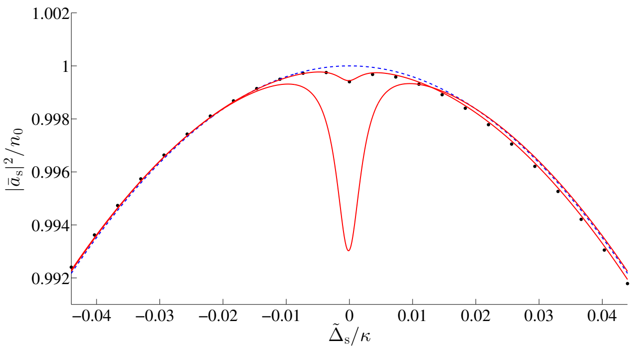

FIG. 3. (color online). The cavity response $|\bar{a}_{\rm s}|^2$ in Eq. (12) in units of $n_0 = (2\Omega_{\rm s}/\kappa)^2$ in the case of pump detuning $\tilde{\Delta}_{\rm p} = -\tilde{\omega}_{\rm m}/2$. The parameters are $G/\omega_{\rm m} = 0.05$, $\kappa/\omega_{\rm m} = 0.05$, $n_{\rm th} = 0$, and $\omega_{\rm m}/\gamma = 10^5$. *Upper solid:* $g/\kappa = 0.1$ and $\Omega_{\rm s}/\kappa = 0.4$. *Lower solid:* $g/\kappa = 0.01$ and $\Omega_{\rm s}/\kappa = 15$. *Dashed:* $g/\kappa = 0$. *Dots:* Numerical results (only available for weak probe drives in our computations). The slight difference for larger $\tilde{\Delta}_{\rm s}$ results from different values of α.

quency, $\tilde{\Delta}_{\rm p} = -\tilde{\omega}_{\rm m}/2$, giving rise to the Hamiltonian (6). Again, we calculate the optical coherence for frequencies close to the cavity resonance frequency, restricting ourselves to the regime $\kappa/\omega_{\rm m} \ll 1$. We find $\langle \hat{a}(t)\rangle = e^{-i\delta t}\bar{a}_{\rm s}$ with

$$\bar{a}_{\rm s} = \bar{a}_{\rm s,0}\left(1 - \alpha - \frac{2g_2^2|\bar{a}_{\rm s,0}|^2\chi_{\rm r}(\tilde{\Delta}_{\rm s})}{\tilde{\gamma}/2 - 2i(\tilde{\Delta}_{\rm s} - \tilde{\Delta}_{\rm p} - \tilde{\omega}_{\rm m}/2)}\right), \quad (12)$$

ignoring a very small constant term. We have defined the coupling rate appearing in Eq. (6) as $g_2 = 8/3\, g(G/\omega_{\rm m})^2$. There is also an OMIT effect in this case, as seen from the last term in Eq. (12), since two probe photons can be converted to one phonon and vice versa. We see that the cavity response is nonlinear in probe power. The dip size for $\tilde{\Delta}_{\rm p} = -\tilde{\omega}_{\rm m}/2$ at $\tilde{\Delta}_{\rm s} = 0$ becomes $d = 4\mu K_2|2\Omega_{\rm s}/\kappa|^2 = 32\mu(g/\kappa)^2(G/\omega_{\rm m})^2|2\Omega_{\rm s}/\kappa|^2$, where the cooperativity is $K_2 = 4g_2^2/(\kappa\tilde{\gamma})$ and the second equality assumes $\tilde{\gamma} \gg \gamma$. The amplitude $|\bar{a}_{\rm s}|^2$ for $\Delta_{\rm p} = -\tilde{\omega}_{\rm m}/2$ is plotted in Fig. 3.

Numerics. To corroborate our analytical results, we numerically solve the quantum master equation (see e.g. Eq. (9) in Ref. [14]) using the system Hamiltonian $\hat{H}(t) = \hat{H}_0 + \hat{H}_1 + i\hbar(e^{-i\delta t}\Omega_{\rm s}\hat{a}^\dagger - \text{h.c.})$ with $\hat{H}_0$ and $\hat{H}_1$ from Eqs. (2) and (3). Since the Hamiltonian only contains the single frequency $\delta = \omega_{\rm s} - \omega_{\rm p}$, we can use the continued-fraction method [40] to solve for the frequency components of the density matrix $\hat{\varrho}(t) = \sum_{n=-N}^{N}\hat{\varrho}_n e^{-in\delta t}$, where N is an integer cutoff. From this, we calculate the part of the optical coherence $\langle\hat{a}\rangle$ rotating at the frequency of the probe drive. Figs. 2 and 3 show that the numerical and analytical calculations are in good agreement.

Conclusion. We have studied corrections to linearized optomechanics and identified signatures of the intrinsic nonlinear coupling between light and mechanical motion. The signatures are nonlinear versions of optomechanically induced transparency, that come about due to resonant two-photon or two-phonon processes in the presence of a strong, off-resonant optical drive. These effects are observable even when the single-photon coupling rate is smaller than the cavity linewidth, and are thus relevant to present day experiments.

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Note added. During the final stages of this project, we became aware of related work by Lemonde, Didier, and Clerk.

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[35] By evaluating the Routh-Hurwitz criterion [43], one finds that the unperturbed system is stable when $G^2 < ((\kappa/2)^2 + \Delta_{\rm p}^2)\omega_{\rm m}/(4|\Delta_{\rm p}|)$, assuming $\Delta_{\rm p} \leq 0$ and $\omega_{\rm m}/\gamma \gg 1$. The assumption $G/\omega_{\rm m} \ll 1$ ensures that this criterion is satisfied for the detunings we discuss here.

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